

VSAQs

ch. 01: FUNCTIONS



- ① If  $A = \{1, 2, 3, 4\}$  and  $f: A \rightarrow \mathbb{R}$  is a function defined by  $f(x) = \frac{x^2 - x + 1}{x + 1}$ , then find the range of  $f$ .
- ② If  $A = \{-2, -1, 0, 1, 2\}$  &  $f: A \rightarrow B$  is a surjection defined by  $f(x) = x^2 + x + 1$ , then find  $B$ .
- ③ Find the domains of the following real valued function  
 $f(x) = \frac{2x^2 - 5x + 7}{(x-1)(x-2)(x-3)}$
- ④ If  $A = \{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$  and  $f: A \rightarrow B$  is a surjection defined by  $f(x) = \cos x$  then find  $B$ .
- ⑤ Find the domain of Real valued function  
 $f(x) = \sqrt{4x - x^2}$
- ⑥  $f(x) = \frac{1}{6x - x^2 - 5}$
- ⑦ If  $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$  by  $f(x) = x^3 - \frac{1}{x^3}$  then S.T  $f(x) + f(\frac{1}{x})$
- ⑧ If  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{1-x^2}{1+x^2}$ , then S.T  $f(\tan 2\theta) = \cos 2\theta$ .
- ⑨ If  $f(x) = 2x - 1$ ,  $g(x) = \frac{x+1}{2} \forall x \in \mathbb{R}$  find  
 (i)  $(g \circ f)(x)$  (ii)  $(f \circ g)(x)$
- ⑩ Find domain of Real valued function  
 $f(x) = \frac{1}{(x^2-1)(x+3)}$
- ⑪  $f(x) = \frac{1}{(x^2-1)(x+3)}$
- ⑫  $f(x) = \log(x^2 - 4x + 3)$

### Ch: 03: Matrices:



13) If  $w$  complex (non-real) cube root of unity, then show that

$$\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$$

14)  $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$  then find

the values of  $x, y, z$  and  $a$ .

15) If  $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$  find  $A^2$

16) If  $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$

17) Find the trace of  $\begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$

18) If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$  find

$3B - 2A$ .

19) If  $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$  &  $A^2 = 0$ , find  $k$

20) Find the rank of matrix  $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

21) If  $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}$  &  $B = \begin{bmatrix} -1 & 1 & 0 \\ -0 & 1 & -2 \end{bmatrix}$  then find  $(AB)^{-1}$

22) Define symmetric matrix.

If  $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$  is a symmetric matrix,

then find  $x$ .

23) If  $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$  is a skew symmetric matrix, then find  $x$ .



24) If  $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$  &  $\det A = 45$  find  $x$ .

25) Define rank of a matrix.

26) Find the rank of matrix  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

27) If  $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & -2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & -10 \\ 2 & 13 \\ 4 & -12 \end{bmatrix}$  and

$X = A+B$  then find  $x$ .

28) Find trace of  $A$ , if  $A = \begin{bmatrix} 1 & 2 & -1/2 \\ 0 & -1 & 2 \\ -1/2 & 2 & 1 \end{bmatrix}$

29) Find  $A^2$ , where  $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$

### Ch:04: ADDITION OF VECTORS

30) If the vectors  $-3i+4j+k$  and  $\mu i+8j+6k$  are collinear vectors, then find  $\lambda$  and  $\mu$ .

31) Find the unit vector in the direction of

vector  $a = 2i+3j+k$

32) Let  $a = i+2j+3k$  &  $b = 3i+j$ , find the unit vector in the direction of  $a+b$ .

33) If  $OA = i + j + k$ ,  $AB = 3i - 2j + k$ ,  $BC = i + 2j - 2k$  and  $CD = 2i + j + 3k$ , then find the vector  $OD$ .

34) If  $a = 2i + 4j - 5k$ ,  $b = i + j + k$  &  $c = j + 2k$  find the unit vector in the opposite direction of  $a + b + c$ .

35) Find the vector equation of the line joining the point  $2i + j + 3k$  &  $-4i + 3j - k$ .

36) Find the v.e of line passing through the point  $2i + 3j + k$  and parallel to the vector  $4i - 2j + 3k$ .

37)  $ABCDE$  is a pentagon. If the sum of vectors  $AB, BC, AE, DC, ED$  and  $AC$  is  $\lambda \cdot AC$ , then find the value of  $\lambda$ .

38) Find the v.e of the plane passing through the points  $i - 2j + 5k, -5j - k, -3i + 5j$ .

### Ch:05: PRODUCT OF VECTORS

39) Find the angle between the vectors  $i + 2j + 3k$  and  $3i - j + 2k$ .

40) For what values of  $\lambda$ , the vectors  $i - \lambda j + 2k$  and  $8i + 6j - k$  are at right angles?

41) If  $\vec{a} = i + 2j + 3k$  &  $\vec{b} = 3i - j + 2k$  then show that  $\vec{a} + \vec{b}$  &  $\vec{a} - \vec{b}$  are perpendicular to each other.

42) Find the angle between planes  $r \cdot (2i - j + 2k)$  and  $r \cdot (3i + 6j + k) = 4$ .

43) If  $a = 2i - 3j + 5k$ ,  $b = -i + 4j + 2k$ , then find  $\vec{a} \times \vec{b}$ .

44) If the vectors  $2i + \lambda j - k$  and  $4i - 2j + 2k$  are perpendicular to each other, find  $\lambda$ .



45) Find the area of parallelogram having  $a = 2j - k$  and  $b = -i + k$  as adjacent sides.

46) Find the eqn of the plane through the point  $(3, -2, 1)$  and  $\perp$  to vector  $(4, 7, -4)$



Ch-06: Trigonometric Ratios upto Transformations

47) Eliminate  $\theta$  from the following

$$x = a \cos^3 \theta, \quad y = b \sin^3 \theta.$$

48) P.T  $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0.$

49) Express  $\frac{1 - \cos \theta + \sin \theta}{1 + \cos \theta + \sin \theta}$  in terms of  $\tan \frac{\theta}{2}$ .

50) Evaluate  $\sin^2 82\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$

51) Prove that  $\sin 78^\circ + \cos 132^\circ = \frac{\sqrt{5}-1}{4}$

52) P.T  $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$

53) Find the value of  $\frac{\sin^2 \frac{\pi}{10} + \sin^2 \frac{4\pi}{10} + \sin^2 \frac{6\pi}{10}}{\sin^2 \frac{9\pi}{10}}$

54) Find the period of  $f(x) = \cos(3x+5) + 7$

55) If  $\sin \theta = \frac{1}{3}$  &  $\theta \in \text{II Quadrant}$  then find  $\cos \theta$

56) If  $a \cos \theta - b \sin \theta = c$  Then S.T  $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$

57) Find the period of the function defined by

$$f(x) = \tan(x + 4x + 9x + \dots + n^2 x) \quad (n \text{ is any positive integer})$$

58) Find a cosine function whose period is 7

59) Find the minimum and maximum values of  $3 \cos x + 4 \sin x$ .

60) Express  $\frac{(\sqrt{3}\cos 25^\circ + \sin 25^\circ)}{2}$  as sine of an angle.

61) What is value of  $\tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ$

62) Simplify  $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right)$

63) Find a sine function whose period is  $\frac{2}{3}$

64) P.T.  $\cos 48^\circ \cos 12^\circ = \frac{3 + \sqrt{5}}{8}$

65) Find the value of  $\sin \frac{5\pi}{3}$

66) Find the period of the function  $\cos\left(\frac{4x+9}{5}\right)$

67) Find the period of the fn  $\tan 5x$ .

### Ch-09: HYPERBOLIC FUNCTIONS

68) P.T.  $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx, x \in \mathbb{R}$

69) S.T.  $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2} \log_e 3$

70) If  $\cosh x = \frac{5}{2}$  find (i)  $\cosh 2x$  (ii)  $\sinh 2x$ .

71) If  $\sinh x = 3$ , S.T.  $x = \log_e(3 + \sqrt{10})$

72) For any  $x \in \mathbb{R}$ , P.T.  $\cosh^4 x - \sinh^4 x = \cosh(2x)$

73) If  $\operatorname{cosech} x = \sec \theta$ , then P.T.  $\tanh^2 \frac{x}{2} = \tan^2 \frac{\theta}{2}$

74) If  $\sinh x = \frac{3}{4}$ , find  $\cosh(2x)$  &  $\sinh(2x)$

75) P.T. for any  $x \in \mathbb{R}$ ,  $\sinh(3x) = 3\sinh x + 4\sinh^3 x$ .

# SAQs

Ch: 03: MATRICES



1) If  $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & -6 \end{bmatrix}$  &  $B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$  - And B

&  $4A - 5B$ .

2) If  $A = \begin{bmatrix} 7 & -2 \\ 1 & 2 \\ 5 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & -1 \\ 4 & 2 \\ -1 & 0 \end{bmatrix}$  then find

$AB^T$  and  $BA^T$

3) If  $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ , s.t.  $AA^T = A^T A = I$

4) s.t.  $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$  is non-singular and find  $A^{-1}$

5) If  $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  find  $A^4$

6) If  $A = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{bmatrix}$  then

find  $3A - 4B^T$

7) If  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  then

s.t.  $(aI + bE)^3 = a^3 I + 3a^2 b E$



8) If  $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$  &  $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0$

then S.T  $abc = -1$

9) Find adj & inverse of  $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$

10) If  $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$  find  $AA^{-1}$

### Ch:04: ADDITION OF VECTORS

11) If  $a, b, c$  are non-coplanar find the point of intersection of the line passing through the points  $2a+3b+c$ ,  $3a+4b-2c$  with the line joining the points  $a-2b+3c$ ,  $a-6b+6c$ .

12) Let  $A, B, C$  and  $D$  be four points with position vectors  $a+2b$ ,  $2a-b$  and  $3a+b$  respectively. Express vectors  $\overline{AC}$ ,  $\overline{DA}$ ,  $\overline{BA}$ ,  $\overline{BC}$ .

13) S.T the pts  $A(2i-j+k)$ ,  $B(i-3j-5k)$ ,  $C(3i-4j-4k)$  are the vertices of right angled triangle.

14) Let  $ABCDEF$  be a regular hexagon with centre 'O'. S.T  $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 6\overline{AO}$ .

15) If the position vectors of the points  $A, B$  and  $C$  are  $-2i+j-k$ ,  $-4i+2j+2k$  and  $6i-3j-13k$  respectively and  $\overline{AB} = \lambda \overline{AC}$  then find the value of  $\lambda$ .

16) If the pts whose position vectors are  $3i-2j-k$ ,  $2i+3j-4k$ ,  $-i+j+2k$  and  $4i+5j+\lambda k$  are coplanar then S.T  $\lambda = \frac{-146}{7}$ .

17) Is the triangle formed by the vectors  $3i+5j+2k$ ,  $2i-3j-5k$  and  $-5i-2j+3k$  coplanar? P.T.

18)  $a, b, c$  are non-coplanar vectors. P.T. the following four pts are coplanar.  $-a+4b-3c$ ,  $3a+2b-5c$ ,  $-3a+8b-5c$ ,  $-3a+2b+c$ .

### Ch: 05: Product of Vectors

19) For any two vectors  $a$  &  $b$ , show that  $(1+|a|^2)(1+|b|^2) = |1-a \cdot b|^2 + |a+b+a \times b|^2$

20) P.T. for any three vectors  $\vec{a}, \vec{b}, \vec{c}$   $[\vec{b}+\vec{c}, \vec{c}+\vec{a}, \vec{a}+\vec{b}] = 2[\vec{a}, \vec{b}, \vec{c}]$ .

21) If  $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$  &  $\vec{b} = 3\vec{i} + 5\vec{j} - \vec{k}$  are two sides of a triangle, then find its area.

22) If  $|a| = 13$ ,  $|b| = 5$  and  $a \cdot b = 60$  then find  $|a \times b|$

23) Let  $\vec{a} = 2\vec{i} + \vec{j} + \vec{k}$  &  $\vec{b} = 3\vec{i} + 4\vec{j} - \vec{k}$ , if  $\theta$  is the angle b/w  $\vec{a}$  &  $\vec{b}$  find  $\sin \theta$ .

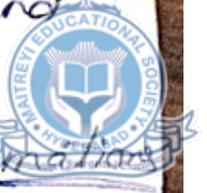
24) If  $a = 2\vec{i} + 3\vec{j} + 4\vec{k}$ ,  $b = \vec{i} + \vec{j} + \vec{k}$  &  $c = \vec{i} - \vec{j} + \vec{k}$  then compute  $a \times (b \times c)$  and verify that it is  $\perp$  to  $a$ .

25) Find unit vector perpendicular to the plane passing through the points  $(1, 2, 3)$ ,  $(2, -1, 1)$  and  $(1, 2, -4)$

26) P.T. angle between any two diagonals of a cube is  $\cos \theta = \frac{1}{3}$

27) If  $\vec{a} = 2\vec{i} + \vec{j} - \vec{k}$ ,  $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$  &  $\vec{c} = \vec{i} + \vec{j} + \vec{k}$  then find  $(a \times b) \cdot (b \times c)$

28) Find the volume of tetrahedron whose vertices are  $(1, 2, 1)$ ,  $(3, 2, 5)$ ,  $(2, -1, 0)$  and  $(-1, 0, 1)$



Ch: 06: Trigonometric Ratios upto Transformation

29) P.T  $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

30) Find the range of  $7\cos x - 24\sin x + 5$

31) If  $\tan 20^\circ = p$  then P.T  $\frac{\tan 610^\circ + \tan 700^\circ}{\tan 560^\circ - \tan 470^\circ} = \frac{1-p^2}{1+p^2}$

32) P.T  $\cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right) = 2$

33) S.T  $\cos^2\left(\frac{\pi}{10}\right) + \cos^2\left(\frac{2\pi}{5}\right) + \cos^2\left(\frac{3\pi}{5}\right) + \cos^2\left(\frac{4\pi}{5}\right) = 2$

34) S.T  $\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right) = \frac{3}{2}$

35) S.T  $(1 + \cos\frac{\pi}{10}) (1 + \cos\frac{3\pi}{10}) (1 + \cos\frac{7\pi}{10}) (1 + \cos\frac{9\pi}{10}) = \frac{1}{16}$

36) P.T  $\cos\frac{2\pi}{7} \cdot \cos\frac{4\pi}{7} \cdot \cos\frac{8\pi}{7} = \frac{1}{8}$

37)  $\cot\left(\frac{\pi}{20}\right) \cot\left(\frac{3\pi}{20}\right) \cot\left(\frac{5\pi}{20}\right) \cot\left(\frac{7\pi}{20}\right) \cot\left(\frac{9\pi}{20}\right) = 1$

38) P.T  $\frac{\cos^3\theta - \cos 3\theta}{\cos\theta} + \frac{\sin^3\theta + \sin 3\theta}{\sin\theta} = 3$

39) If  $\sin d + \operatorname{cosec} d = 2$ , find the value of

$\sin^n d + \operatorname{cosec}^n d, n \in \mathbb{Z}$

40) If 'A' is not an integral multiple of  $\pi$   
P.T  $\cos A \cdot \cos 2A \cdot \cos 4A \cdot \cos 8A = \frac{\sin 16A}{16 \sin A}$  & hence

deduce that  $\cos\frac{2\pi}{15} \cos\frac{4\pi}{15} \cos\frac{8\pi}{15} \cos\frac{16\pi}{15} = \frac{1}{16}$

Ch:07: TRIGONOMETRIC EQUATIONS



41) Solve  $7\sin^2\theta + 3\cos^2\theta = 4$

42) Solve  $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

43)  $\sin x + \sqrt{3}\cos x = \sqrt{2}$

44) If  $\theta_1, \theta_2$  are the solutions of the eqn

$a\cos 2\theta + b\sin 2\theta = c$ , where  $a, b, c \in \mathbb{R}$  and if

$a^2 + b^2 > 0$ ,  $\cos \alpha \neq \cos \beta$  &  $\sin \alpha \neq \sin \beta$ . Then S.T

$\sin \alpha + \sin \beta = \frac{2bc}{a^2 + b^2}$ .

45) Solve the equation  $2\cos^2\theta + 11\sin\theta = 7$

& write the general solutions.

Ch:08: Inverse Trigonometric Functions

46) Find the values of  $\sin(\cos^{-1}(\frac{3}{5}) + \cos^{-1}(\frac{12}{13}))$

47) P.T  $\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

48) P.T  $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{36}{85}$

49) P.T  $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{8}$

50) P.T  $\cos^{-1}\frac{4}{5} + \sin^{-1}\frac{3}{\sqrt{34}} = \tan^{-1}\frac{27}{11}$

Ch:10: Properties of Triangles

51) In a  $\Delta ABC$  S.T  $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$

52) If a  $\Delta ABC$  if  $a:b:c = 7:8:9$  then find  $\cos A : \cos B : \cos C$ .

$$53) \text{ P.T } b^2 \cos^2 \frac{C}{2} + c^2 \cos^2 \frac{B}{2} = s^2$$

$$54) \text{ P.T } \cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$$



$$55) \text{ P.T } \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{s^2}{\Delta}$$

$$56) \text{ P.T } \tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2} = \frac{bc + ca + ab - s^2}{\Delta}$$

57) If  $a = 2\text{cm}$ ,  $b = 3\text{cm}$ ,  $c = 4\text{cm}$  then find  $\cos A$

$$58) \text{ P.T } r(r_1 + r_2 + r_3) = ab + bc + ca - s^2$$

$$59) \text{ P.T } r_1^2 + r_2^2 + r_3^2 + r^2 = 16R^2 - (a^2 + b^2 + c^2)$$

$$60) \text{ If } a = (b+c)\cos\theta, \text{ then P.T } \sin\theta = \frac{2\sqrt{bc}}{b+c} \cos \frac{A}{2}$$

$$61) \text{ In } \Delta ABC, \text{ P.T } \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{1}{r}$$



Ch: 01: FUNCTIONS

1) Let  $f: A \rightarrow B$ ,  $I_A$  and  $I_B$  be identity functions of  $A$  and  $B$  respectively.

P.T  $f \circ I_A = f = I_B \circ f$ .

2) If  $f: R \rightarrow R$ ,  $g: R \rightarrow R$  are defined by  $f(x) = 4x - 1$  &  $g(x) = x^2 + 2$  then find

(i)  $(g \circ f)(x)$  (ii)  $(g \circ f)\left(\frac{a+1}{4}\right)$

(iii)  $f \circ f(x)$  (iv)  $g \circ (f \circ f)(0)$

3) If  $A = \{1, 2, 3\}$ ,  $B = \{a, b, g\}$ ,  $C = \{p, q, r\}$  and  $f: A \rightarrow B$ ,  $g: B \rightarrow C$  are defined by

$f = \{(1, \alpha), (2, \alpha), (3, \beta)\}$ ,  $g = \{(a, q), (b, p), (g, r)\}$  then s.t.  $f$  &  $g$  are bijective functions and  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ .

4) If  $f: A \rightarrow B$  is a bijection &  $g: B \rightarrow C$  is also a bijection then P.T  $g \circ f: A \rightarrow C$  is a bijection.

5) If  $f: A \rightarrow B$  is a bijection,  $I_A$  and  $I_B$  are identity fns on  $A$  &  $B$  respectively, then s.t.  $f \circ f^{-1} = I_B$  &  $f^{-1} \circ f = I_A$ .

6) If  $f: A \rightarrow B$ ,  $g: B \rightarrow C$  are bijections then P.T  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ .

7) If  $f = \{(4, 5), (5, 6), (6, -4)\}$ , and

$g = \{(4, -4), (6, 5), (8, 5)\}$  then find

(i)  $f \circ g$  (ii)  $2f + 4g$  (iii)  $f + g$  (iv)  $f \circ f$

(v)  $\sqrt{f}$  (vi)  $f^2$  (vii)  $|f|$  (viii)  $f - g$

(ix)  $f \circ g$

## Ch:02: MATHEMATICAL INDUCTION

8) Using Mathematical induction, prove the following statement for all  $n \in \mathbb{N}$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

9) S.T  $\forall n \in \mathbb{N}$ ,  $\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots$  upto  $n$  terms  $= \frac{n}{3n+1}$

10) S.T  $(4 \cdot 9^n + 16n - 1)$  is divisible by 64 for all positive integers  $n$

11) P.T  $(1 + \frac{3}{1})(1 + \frac{5}{4})(1 + \frac{7}{9}) \dots (1 + \frac{2n+1}{n^2}) = (n+1)^2 \forall n \in \mathbb{N}$

12)  $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \dots$  upto  $n$  terms  $= \frac{n(n+1)(n+2)(n+3)}{4}$

13)  $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$



14) S.T. 
$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a) \\ (ab+bc+ca)$$

15) solve the eqns by using Gauss Jordan method

$$x - y + 3z = 5, \quad 4x + 2y - z = 0, \quad -x + 3y + z = 5$$

16) Solve by using Matrix Inversion method

$$3x + 4y + 5z = 18, \quad 2x - y + 8z = 13 \quad \text{and} \quad 5x - 2y + 7z = 20$$

17) Solve the following Simultaneous linear eqns by using cramer's & matrix inversion methods

(i)  $x - y + 3z = 5, \quad 4x + 2y - z = 0, \quad -x + 3y + z = 5$

(ii)  $x + y + z = 1, \quad 2x + 2y + 3z = 6, \quad x + 4y + 9z = 3$

18) Solve by using Cramer's rule

(i)  $2x - y + 3z = 8, \quad -x + 2y + z = 4, \quad 3x + y - 4z = 0$

19) S.T. 
$$\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ a & b & c \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

20) S.T. 
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

21) If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$  then S.T.  $A^2 - 4A - 5I = 0$

22) S.T the points  $(5, -1, 1)$ ,  $(7, -4, 7)$ ,  $(1, -6, 10)$  and  $(-1, -3, 4)$  are the vertices of a rhombus by vectors.



23) If  $\vec{a} = i - 2j + k$ ,  $\vec{b} = 2i + j + k$ ,  $\vec{c} = i + 2j - k$ , find  $\vec{a} \times (\vec{b} \times \vec{c})$  and  $|(\vec{a} \times \vec{b}) \times \vec{c}|$ .

24) If  $\vec{a} = i - 2j + 3k$ ,  $\vec{b} = 2i + j - k$  and  $\vec{c} = i + 3j - 2k$ , Verify that  $\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$ .

25) If  $\vec{a} = i - 2j + 3k$ ,  $\vec{b} = 2i + j + k$ ,  $\vec{c} = i + 2j + 2k$ , then find  $(\vec{a} \times \vec{b}) \times \vec{c}$  and  $|\vec{a} \times (\vec{b} \times \vec{c})|$ .

26) If  $\vec{a} = 2i + j - 3k$ ,  $\vec{b} = i - 2j + k$ ,  $\vec{c} = -i + j - 4k$  and  $\vec{d} = i + j + k$  then compute  $|(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})|$ .

27) If  $\vec{a} = 7i - 2j + 3k$ ,  $\vec{b} = 2i + 8k$  and  $\vec{c} = i + j + k$  compute  $\vec{a} \times \vec{b}$ ,  $\vec{a} \times \vec{c}$ ,  $\vec{a} \times (\vec{b} + \vec{c})$ .

then verify whether the cross product is distributive over vector addition.

## Ch:06: Trigonometric Ratios upto Transformations

28) If  $A, B, C$  are angles in a triangle, then  $\cos 2A - \cos 2B + \cos 2C = 1 - 4 \sin A \cos B \sin C$ .

(29) If  $A, B, C$  are the angles of a triangle,  
 P.T  $\sin^2 A + \sin^2 B + \sin^2 C = 4 \sin A \sin B \sin C$

(30) If  $A, B, C$  are angles in a triangle then  
 P.T  $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

(31) P.T  $\sin^2 A - \sin^2 B + \sin^2 C = 4 \cos A \sin B \cos C$

(32)  $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$  (i)

(33)  $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$  (ii)

(34)  $\sin A + \sin B - \sin C = 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$  (iii)

(35) If  $A + B + C = 2S$ , then P.T

$$\sin(S-A) + \sin(S-B) + \sin C = 4 \cos \frac{S-A}{2} \cos \frac{S-B}{2} \sin \frac{C}{2}$$

(36) If  $A + B + C = 2S$ , then P.T

$$\begin{aligned} \cos(S-A) + \cos(S-B) + \cos(S-C) &= 4 \cos \frac{S-A}{2} \cos \frac{S-B}{2} \cos \frac{S-C}{2} \\ &= 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} \end{aligned}$$

### Ch:10 : PROPERTIES OF TRIANGLES

(37) In  $\Delta ABC$ , if  $AD, DE, CF$  are the perpendicular drawn from the vertices  $A, B, C$  to the opposite sides, S.T

(i)  $\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{1}{r}$

(ii)  $AD \cdot BE \cdot CF = \frac{(abc)^2}{8R^3}$

38) If  $\sin \theta = \frac{a}{b+c}$ , then S.T  $\cos \theta = \frac{2\sqrt{bc}}{b+c}$  W.A



39) If  $p_1, p_2, p_3$  are altitudes drawn from vertices A, B, C to the opposite sides of  $\Delta$  respectively S.T

(i)  $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{a}$

(ii)  $\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{1}{a_3}$

(iii)  $p_1 p_2 p_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$

39) If  $a=13, b=14, c=15$ . S.T  $R = \frac{65}{8}$ ,  
 $r=4, r_1 = \frac{21}{2}$  &  $r_3=14, r_2=12$

40) If  $r:R:r_1 = 2:5:12$ , then P.T the triangle is right angled at A.

41) P.T  $4(r_1 r_2 + r_2 r_3 + r_3 r_1) = (a+b+c)^2$

42) If  $r_1=2, r_2=3, r_3=6$  &  $r=1$  P.T  
 $a=3, b=4, c=5$

43) P.T  $\frac{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}}{\cot A + \cot B + \cot C} = \frac{(a+b+c)^2}{a^2+b^2+c^2}$

44) In  $\Delta ABC$ , if  $r_1=8, r_2=12, r_3=24$  find  $a, b, c$ .